Solution to Sample Quiz 2

ENVIRON 805K

November 16, 2017

1.

(a)

When there is no correction for the externality, $P^s = MPC = 2 + Q$. Let $P^d = P^s$, we have $Q^e = 11$ and $P^e = 13$.

(b)

In terms of the social optimum, we should take the externalities into consideration. The marginal social cost is $MSC = MPC + Q = 2 + 2Q$. In this case, $P^s = MSC = 2 + 2Q$. Let $P^s = P^d$, we have $Q^* = \frac{22}{3}$.

(c)

See Figure 1, the deadweight loss due to the externality is $S_\Delta ABC = \frac{121}{6}$.

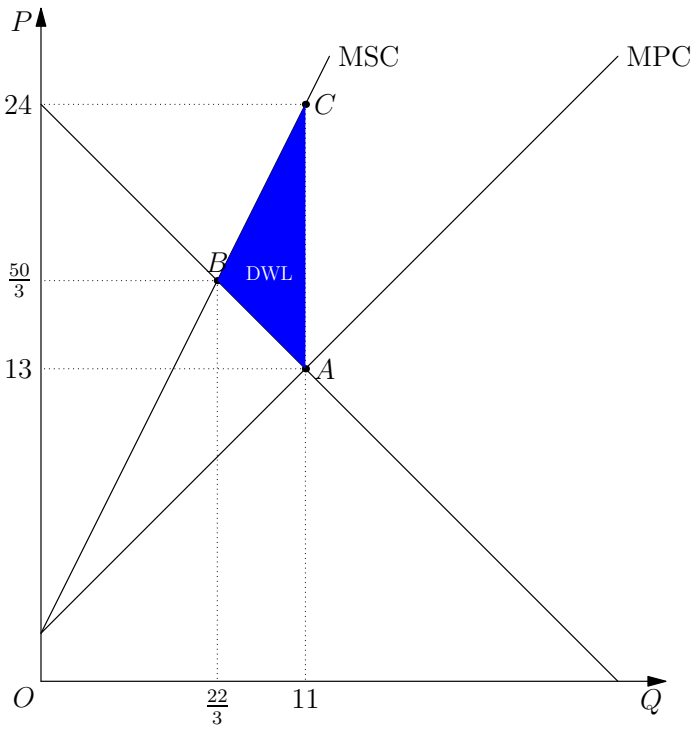

Figure 1: Private Supply and Social Optimum in Competitive Market

(d)

The emission fee should equal the marginal cost that inflicts the residents. Therefore, $T = 1$.

(e)

If the industry is a monopoly, marginal cost is equal to marginal revenue.

$$MR = \frac{d(P \times Q)}{dQ} = 24 - 2Q$$

Let $MR = MPC$, we have $P^e = \frac{28}{3}$ and $Q^e = \frac{22}{3}$. Taking pollution into consideration, it is exactly the social optimum. Therefore, there is no deadweight loss.

2.

See Figure 2.

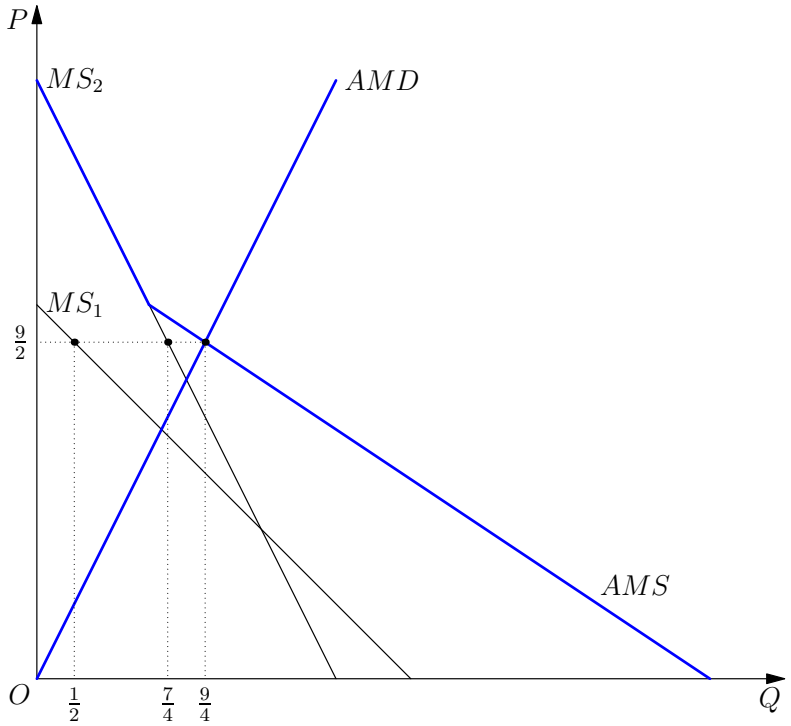

Figure 2: Two Polluters and Two Victims

(a)

$MS_1(e) = -MAC_1(e) = 5 - e$

$MS_2(e) = -MAC_2(e) = 8 - 2e$

The aggregate saving function is $AMS(e) = \begin{cases} 8 - 2e, 0 \leq e \leq \frac{3}{2} \\ 6 - \frac{2}{3}e, e \geq \frac{3}{2} \end{cases}$.

(b)

The aggregate marginal damage is $AMD(e) = 2e$.

(c)

Let $AMD = AMS$, we have $e^* = \frac{9}{4}$ and $p^* = \frac{9}{2}$.

(d)

$e_1^* = \frac{1}{2}$ and $e_2^* = \frac{7}{4}$.

3.

Recall that firms will operate on the portion of their marginal cost curve that lies above the average variable cost. Therefore, we need to verify this condition after deriving the output.

(a)

Firm	**VC**	**AVC**	**MC**
Fireyear	$300 + 2Q_F^2$	$\frac{300}{Q_F} + 2Q_F$	$4Q_F$
Goodstone	$500 + Q_G^2$	$\frac{500}{Q_G} + Q_G$	$2Q_G$

Let $P = MC_F$, we have $Q_F = 15$. Since $MC_F > AVC_F$, it will keep operating. The profit for Fireyear is $\Pi_F = P \times Q_F - VC_F - FC_F = 150$.

Let $P = MC_G$, we have $Q_G = 30$. Since $MC_G > AVC_G$, it will keep operating. The profit for Goodstone is $\Pi_G = P \times Q_G - VC_G - FC_G = 400$.

(b)

Since the marginal damage from pollution is 12 per ton, the Pigovian tax for per unit of emissions is $T = 12$.

Firm	**VC**	**AVC**	**MC**
Fireyear	$300 + 2Q_F^2 + 12Q_F$	$\frac{300}{Q_F} + 2Q_F + 12$	$4Q_F + 12$
Goodstone	$500 + Q_G^2 + 12Q_G$	$\frac{500}{Q_G} + Q_G + 12$	$2Q_G + 12$

Let $P = MC_F$, we have $Q_F = 12$. Since $MC_F < AVC_F$, it will shut down.

Let $P = MC_G$, we have $Q_G = 24$. Since $MC_G > AVC_G$, it will keep operating. The profit for Goodstone is $\Pi_G = P \times Q_G - VC_G - FC_G = 76$.

(c)

The subsidy for per unit of pollution abated is $T = 12$. For Fireyear, the total subsidy is $12(15 - Q_F)$. For Goodstone, the total subsidy is $12(30 - Q_G)$.

Firm	**VC**	**AVC**	**MC**
Fireyear	$300 + 2Q_F^2 - 12(15 - Q_F)$	$\frac{120}{Q_F} + 2Q_F + 12$	$4Q_F + 12$
Goodstone	$500 + Q_G^2 - 12(30 - Q_G)$	$\frac{140}{Q_G} + Q_G + 12$	$2Q_G + 12$

Let $P = MC_F$, we have $Q_F = 12$. Since $MC_F > AVC_F$, it will keep operating. The profit for Goodstone is $\Pi_F = P \times Q_F - VC_F - FC_F = 168$.

Let $P = MC_G$, we have $Q_G = 24$. Since $MC_G > AVC_G$, it will keep operating. The profit for Goodstone is $\Pi_G = P \times Q_G - VC_G - FC_G = 436$.

4.

See Figure 3.

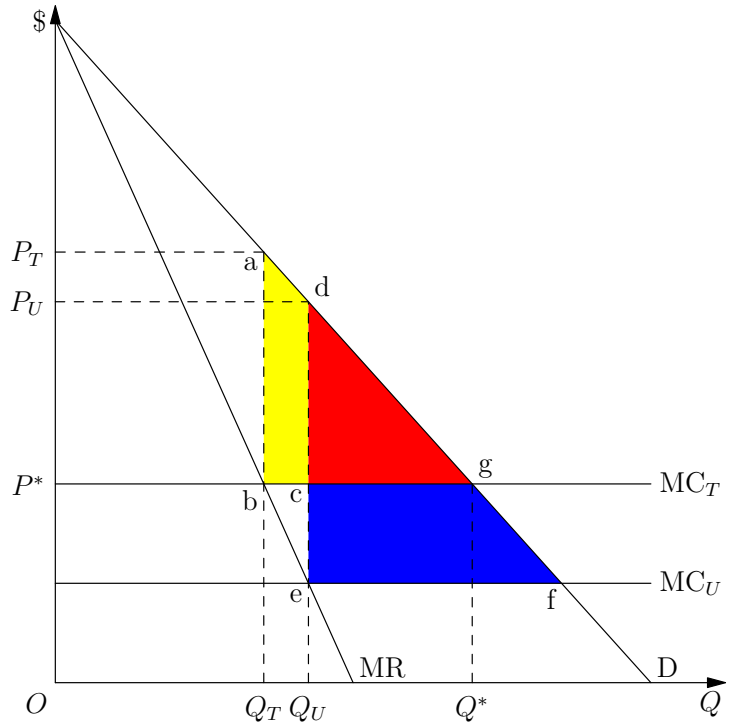

Figure 3: Monopoly and Pigovian Tax

(a)

$MR = \frac{dP \times Q}{dQ} = 2 - Q$. Let $MR = MPC$, the unregulated steel output level is $Q_U = 1.7$. The price is $P_U = 1.15$. In this case, the social optimum is point f. The deadweight loss is $S_{def} = 0.7225$.

(b)

Notice that the social optimum is f when we ignore the pollution. In this case, the marginal social cost is $MSC = MPC + MD = 0.6$. Let $D = MSC$, the social optimum under pollution is g. Therefore, the socially optimal output of steel is $Q^* = 2.8$. The deadweight loss from unregulated monopolist is $S_{dcg} = 0.3025$.

(c)

With a Pigovian tax, the marginal private cost is $MPC_T = 0.6$. Let $MPC_T = MR$, we can derive the steel output level with Pigovian tax $Q_T = 1.4$ and $P_T = 1.3$.

(d)

The extra deadweight loss from Pigovian fee is $S_{abcd} = 0.1875$.

(*Hint: In order to help you better understand this problem, I add three figures in the end to show the three cases for sub-question (a), (b), and (c). See Figure 5, 6, and 7.*)

5.

(a)

In this case, we need to maximize the total profit of the two companies.

$$\max_{B,S} \quad \Pi(B,S) = 10B + 14S - (B^2 + BS + 4) - (S^2 + 8)$$

The first order conditions are

$$\begin{cases} \frac{\partial \Pi}{\partial B} = 10 - 2B - S = 0 \\ \frac{\partial \Pi}{\partial S} = 14 - B - 2S = 0 \end{cases}$$

Solving the equations, we have $B^* = 2$ and $S^* = 6$.

Before answer (b) and (c), let's first calculate the output for each company when the other one does not operate.

For the shoe company

$$\max_S \quad \Pi_S(S) = 14S - (S^2 + 8)$$

The first order condition is $\frac{\partial \Pi_S}{\partial S} = 14 - 2S = 0$. Therefore, $S = 7$.

For the bakery company

$$\max_B \quad \Pi_B(B, 0) = 10B - (B^2 + 4)$$

The first order condition is $\frac{\partial \Pi_B}{\partial B} = 10 - 2B = 0$. Therefore, $B = 5$.

(b)

If Finch's Footwear has rights to pollute, let us consider the profit the bakery company.

- If both of them operate, the profit of the bakery company is $\Pi_B(2, 6) - (\Pi_S(7) - \Pi_S(6)) = -1$
- If the bakery company shuts down, its profit is 0.
- If the shoes company shuts down, the profit for the bakery company is $\Pi_B(5, 0) - \Pi_S(7) = -20$.

Therefore, the bakery company will shut down and the shoes company will produce 7 shoes.

(c)

If Millie's Muffins has a right to clean air, let us consider the profit of the shoes company.

- If both of them operate, the profit of the shoes company is $\Pi_S(6) - (\Pi_B(5, 0) - \Pi_B(2, 6)) = 19$
- If the bakery company shuts down, the profit for the shoes company is $\Pi_S(7) - \Pi_B(5, 0) = 20$.
- If the shoes company shuts down, its profit is 0.

Therefore, the bakery company will shut down and the shoes company will produce 7 shoes.

6.

See Figure 4. No matter what kind of regulation is, the deadweight loss is the red and blue area.

Take the high-damage type as an example. The optimal level should be B. If we set an emission control of e^*, the deadweight loss is the blue area. If we charge a pollution fee p^*, the firm will emit e^*, and therefore the deadweight loss is also the blue area.

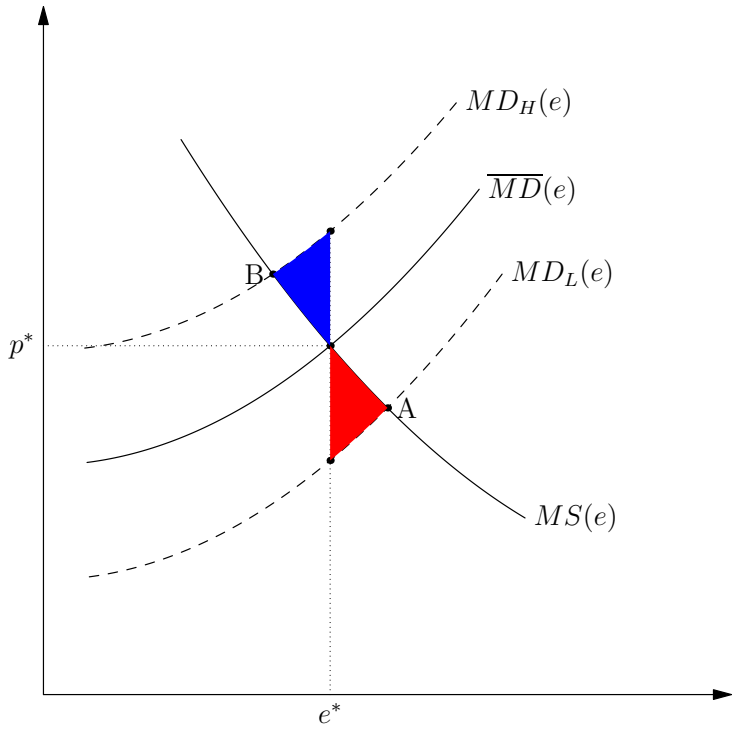

Figure 4: Fees and Quantity Regulations under Unknown Marginal Damage

Appendix

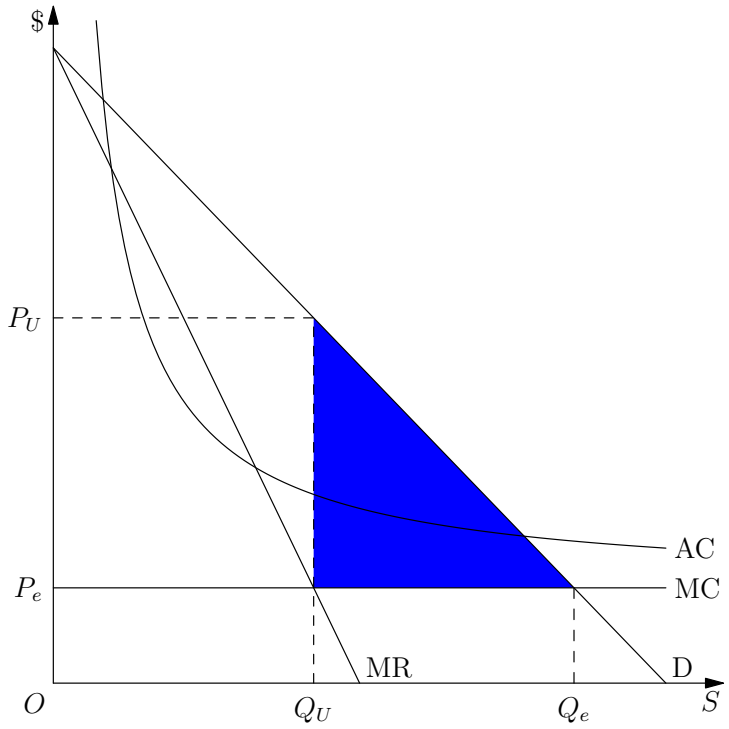

Figure 5: Monopoly

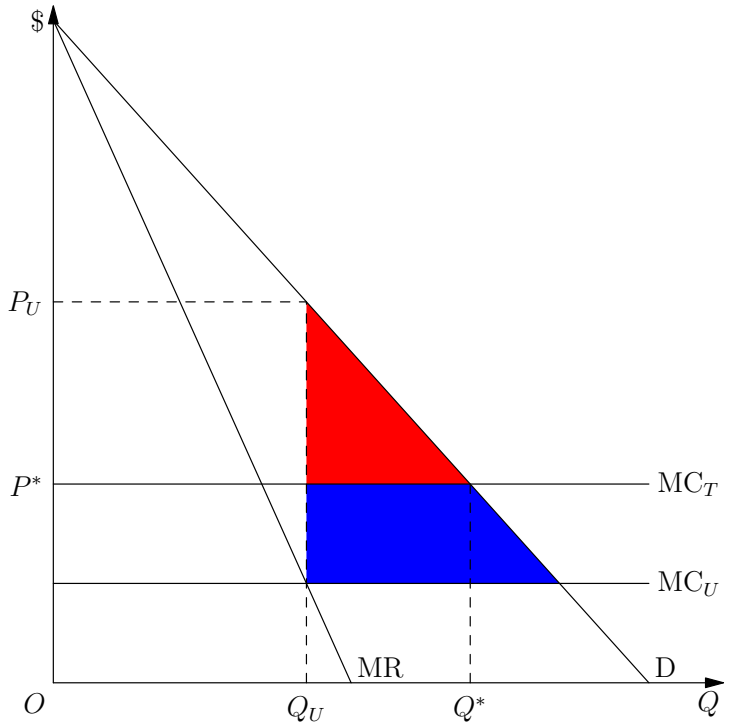

Figure 6: Monopoly with Pollution

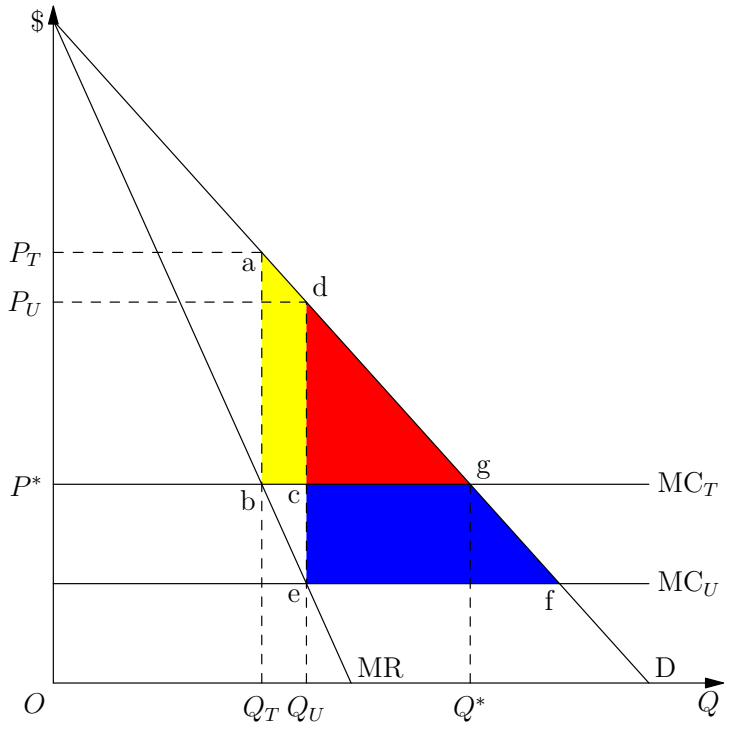

Figure 7: Monopoly with Pigovian Tax