Solution to Problem Set 2

ENVIRON 805K

October 9, 2017

1. (3.6)

(a)

Biocentrism views humans as just another species with no special claim on the world's resources. It emphasizes that all living things have intrinsic value, regardless of their instrumental value. Therefore, Biocentrism mainly focuses on the environment health H.

In contrast, antropocentrism believes the environment exists only to serve people's material needs. (Note: We use the more restrictive definition of anthropocentrism to include only material gain from the environment.) Therefore, anthropocentrism mainly focuses on commodity consumption.

According to the utility function, environmental health and consumption are both determinants of individual's utility, which means people care about the intrinsic value of the environment. Therefore, the utility function is not consistent with pure biocentrism or anthropocentrism. It is more consistent with utilitarianism.

(b)

Assume $g = 0.1$ and $n = 1$, we have $H_t = 0.9H_{t-1} + 10 - x$. Now we solve the expression of series $\{H_t\}$.

$$
\begin{aligned}
H_t &= 0.9H_{t-1} + 10 - x \\
\Rightarrow H_t + 10x - 100 &= 0.9(H_{t-1} + 10x - 100) \quad (*) \\
\Rightarrow H_t + 10x - 100 &= 0.9^t(H_0 + 10x - 100) \\
\Rightarrow H_t + 10x - 100 &= 0.9^t \cdot 10x \\
\Rightarrow H_t &= (0.9^t - 1) \cdot 10x + 100
\end{aligned}
$$

(*Hint: Using the method of undetermined coefficients, we can obtain equation (*).*)

Figure 1 depicts the evolution of H.

(*Note: Actually, you don't need to solve the expression of $\{H_t\}$. You can conduct simulation using different values of t in a statistical software and plot the figure.*)

(c)

Based on the result of (b), we have

$$U_t(x, H_t) = x + H_t - \frac{1}{x} - \frac{1}{H_t} = x + (0.9^t - 1) \cdot 10x + 100 - \frac{1}{x} - \frac{1}{(0.9^t - 1) \cdot 10x + 100}$$

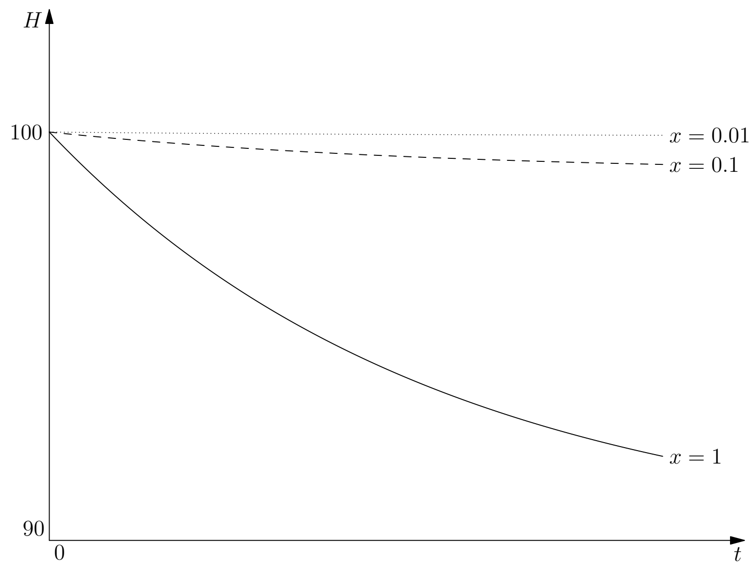

Figure 1: The Evolution of H

Figure 2 depicts the evolution of U considering different values of x (Here, I take the value of x from 0.05 to 1.1, and the interval is 0.05.). The blue curve is the simulation using $x = 0.334$ while the yellow curves using other values of x. According to this figure, we can find that the optimal consumption level is about 0.334. This is because at this level, individual's utility is maximized in the long term. (It is OK if your answer is about 0.3.)

However, the conclusion above is a little hasty since we only know limited information. In order to obtain the accurate optimal consumption level, the discount rate, individual's life cycle, and many other extra conditions are also needed here.

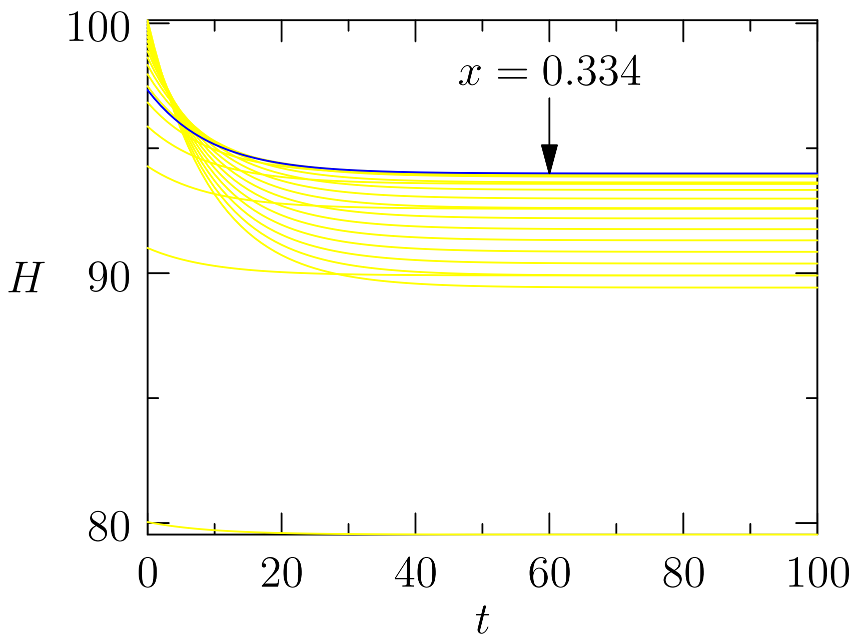

Figure 2: The Evolution of U

Extension:

(The content in this part is optional for you. You don't need to read this if you are not interested.)

Now, let us think further: why this result will come up that the optimal consumption level is about 0.334? Let's focus on the utility function.

$$\frac{\partial U_t}{\partial x} = 1 + 10(0.9^t - 1) + \frac{1}{x^2} + \frac{10(0.9^t - 1)}{\left[(0.9^t - 1)\cdot 10x + 100\right]^2}$$

Let $\frac{\partial U_t}{\partial x} = 0$ and since $lim_{t\to+\infty}(0.9^t - 1) = -1$, we have

$$\begin{aligned}\frac{\partial U_t}{\partial x} = -9 + \frac{1}{x^2} - \frac{10}{(100-10x)^2} = 0 \\ \Leftrightarrow (90x^2 - 10)(10-x)^2 + x^2 = 0\end{aligned}$$

According to Figure 3, the solution is $x = 0.334$. Therefore, the optimal consumption level

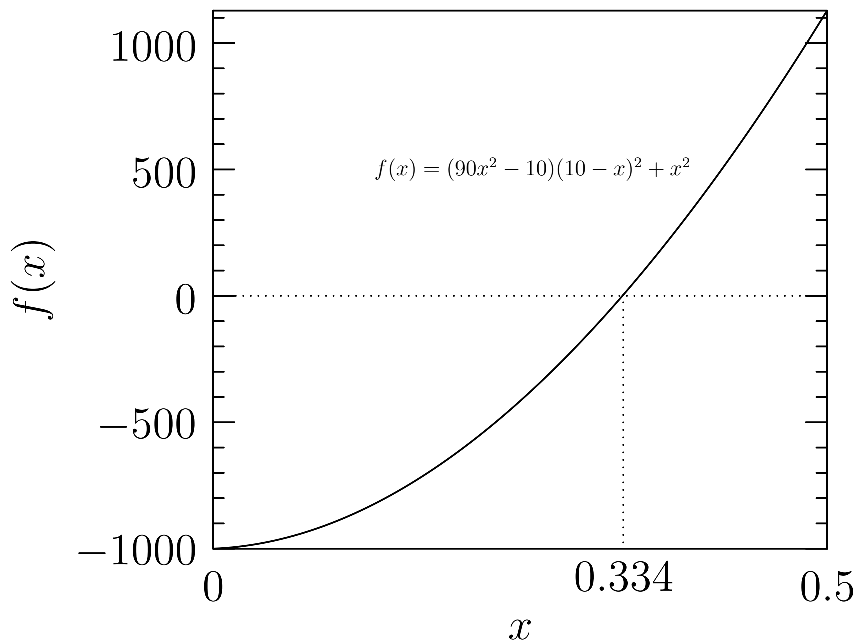

Figure 3: $f(x) = (90x^2 - 10)(10 - x)^2 + x^2$

is 0.334.

2. (3.7)

(a)

Figure 4 shows the indifference curve for Tucker and Finch. Since the indifference curves are continuous, convex, and downward-sloping, individual preferences are not problematic. (Actually, the red points should be above the curves while the blue points should be under the curves. However, I tried my best...)

(b)

Tucher and Finch will prefer B. This is because $(1.7, 1.3) \succ_T (2.0, 1.0)$ and $(1.8, 1.1) \succ_F (2.0, 1.0)$, and therefore each individual is better off in state B.

(c)

There are a total of 4 units of food and 2 units of sport in state A. Suppose we redistribute the quantities in state A as follows: assign $(2.4, 0.7)$ to Tucker and $(1.6, 1.3)$ to Finch. Then both individuals are better off than in state B.

(d)

Compensation principal states that winners have to compensate losers for the social choice judgment to be valid. However, it is possible for every individual to be better off in one state than another. In the example showing in (c), actually both Tucker and Finch are better off after the redistribution. There is no winners or losers, which challenges the claim that "winners have to compensate losers".

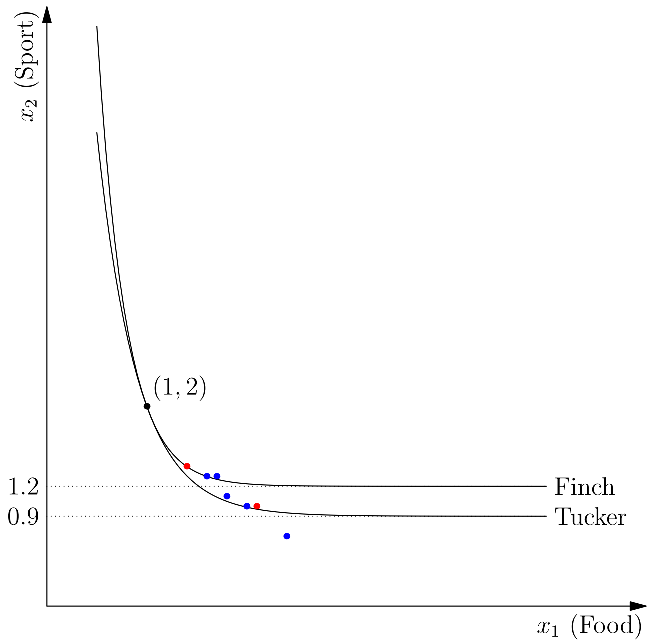

Figure 4: Indifference Curves for Tucker and Finch

(Note: You can refer to Kenneth J. Arrow, *Social Choice and Individual Values*, Second Edition) for more details.

3. (4.6)

According to the *Equimarginal Principal*: In controlling emissions from several polluters, all emitting the same pollutant, efficiency requires that the marginal cost of emission control be the same for all polluters. Now that the polluter's cost of pollution control differs widely from each other, their marginal cost of emission control cannot be the same. Therefore, this is inefficient.

Let's look at an example. Suppose there are two polluters (A and B). If polluter A has a higher marginal cost of emission control than B, it can pay the same money to B and ask B to reduce the pollution emission. Then, the total cost stays constant while there will be more pollution emission reduction. This is a Pareto improvement.

4. (4.7)

Figure 5 shows the Edgeworth box for Matilda and Humphrey and their smoke and rental payments. The total amount of rental payments is the length of the horizontal axis while the total amount of smoke is the length of the vertical axis. In particular, A represents the initial endowment: Humphrey smokes 20 packs of cigarettes and they split the rental expense 1:1. Any allocation in the shaded area is a Pareto improvement.

Let the price of house rent be p_h and suppose Matilda will pay p_c for each pack of cigarettes. Let $E^*(h^*, c^*)$ be house rent and smoke reduction that Matilda ends up with. Then, Matilda's budget constraint for any (h, c) is

$$\frac{1}{2}p_h = p_h h + p_c c$$
$$\Leftrightarrow c = -\frac{p_h}{p_c}h + \frac{p_h}{2p_c}$$

At the equilibrium E^*, the price must satisfies two requirements: (1) The budget line AB must pass through the initial endowment; (2) The budget line is tangent to both Hamphrey and Matilda's indifference curve. Suppose the slope of AB is k, then we have

$$k = -\frac{p_h}{p_c}$$

Therefore

$$p_c = -\frac{p_h}{k}$$

The situation where there is no smoking in the beginning is similar to the analysis above.

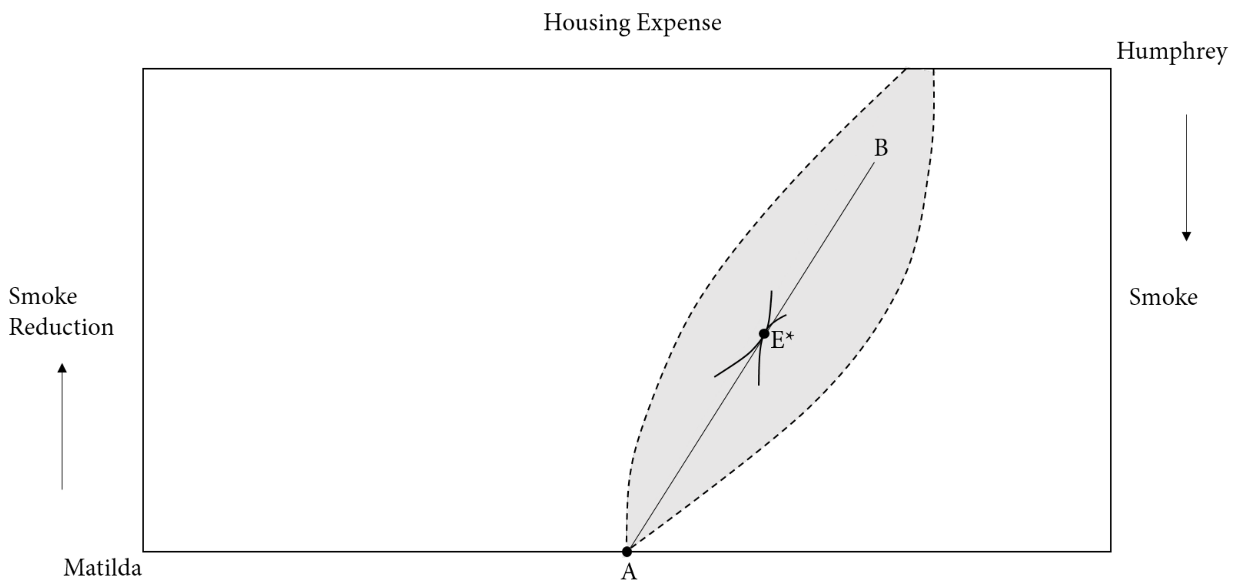

Figure 5: Edgeworth Box

5. (5.2)

(a)

Fritz's total cost is

$$C(d) = \frac{1}{d^2} + 2d \quad (d \geq 0.1)$$

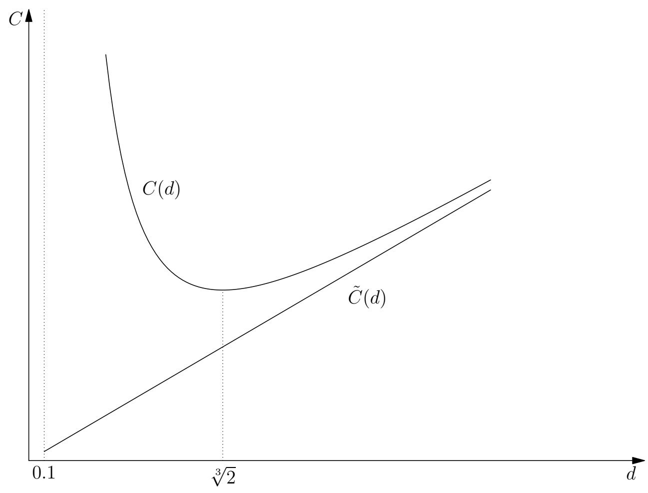

Figure 6: Fritz's Total Cost at Distance d

(b)

The first order condition is

$$C'(d) = -\frac{2}{d^3} + 2 = 0$$

Therefore, the optimal distance Fritz will live from the airport is

$$d^* = 1$$

His total costs are

$$C(1) = 3$$

(c)

If Fritz is compensated for his damage, his cost is

$$\tilde{C}(d) = 2d \quad (d \geq 0.1)$$

Therefore, he will choose to live 0.1km away from the airport. He will be compensated $\frac{1}{0.1^2} = 100$ dollars.

6. (5.5)

(a)

The aggregate marginal damage for the public bad is $8p$.

(b)

See Figure 7.

(c)

In the absence of any regulation or bargaining, the cheese factory will maximize its total saving. Therefore it will pollute at the level where the marginal saving is 0, i.e., $p = 10$.

For the whole society, the externalities of the pollution should be considered. Therefore, at the optimal level, marginal savings should equal the aggregate marginal damage.

$$20 - 2p = 8p$$
$$\Rightarrow p = 2$$

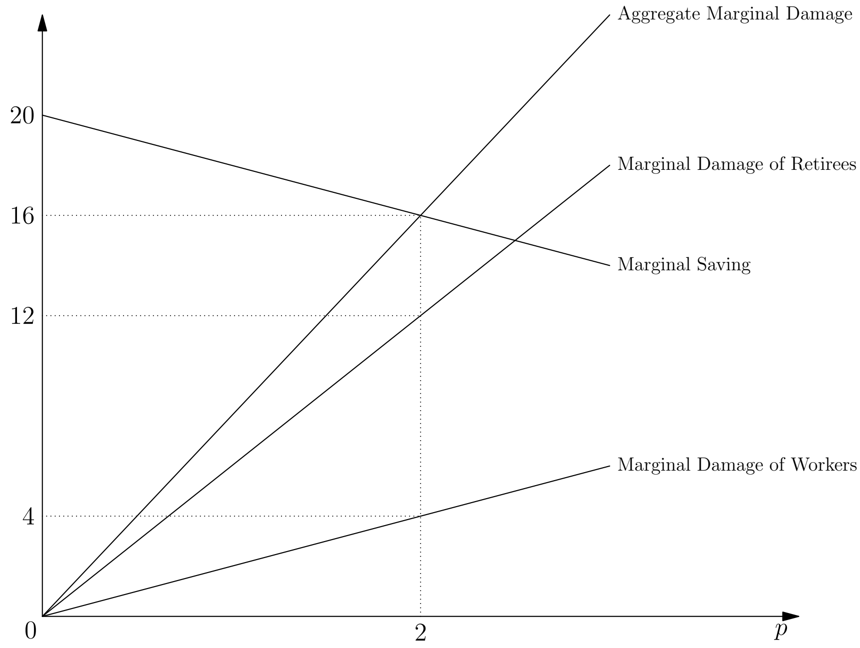

Figure 7: Marginal Savings and Marginal Damages

(d) When the pollution abatement is A, the workers and the retirees will get benefit from damage reduction.

For the workers, the damage reduction is $10^2 - (10 - A)^2 = -A^2 + 20A$, and therefore their MWTP is $-2A+20$. For the retirees, the damage reduction is $3 \times 10^2 - 3(10-A)^2 = -3A^2 + 60A$, and therefore their MWTP is $-6A + 60$. The aggregate MWTP is $-8A + 80$.

(e)

When firm reduce its pollution by A, its saving will decrease by $20 \times 10 - 10^2 - \big(20(10 - A) - (10 - A)^2\big) = A^2$. Therefore, its marginal cost of pollution abatement is $2A$.

At the optimal level of A, firm's marginal cost of pollution abatement should equal the aggregate MWTP. That is

$$2A = -8A + 80$$
$$\Rightarrow A = 8$$

(f)

Based on the results of (c) and (e), the optimal level of pollution is the same, that is $p = 2$. Therefore, these problems are equivalent. This is because they are two aspects of a same problem, and end up with a same condition. Our goal is to choose a condition that optimize the social welfare. Therefore, we will obtain the same answer from these problems.

7. (5.6)

(a) At the socially efficient level, the marginal abatement cost should equal the total MWTP (See Figure 8). Therefore, the emission reduction should be 5.

(b)

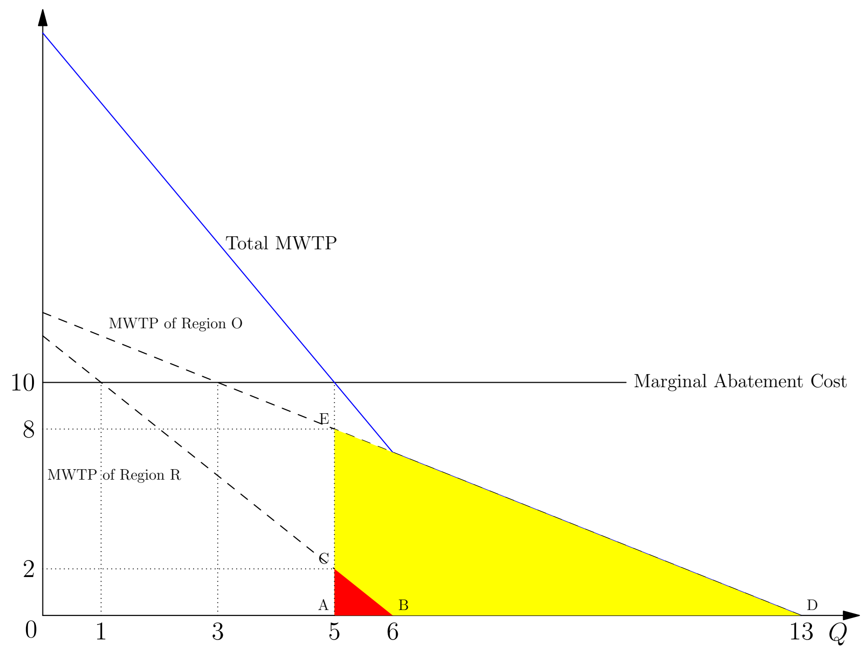

Figure 8: Marginal Willingness to Pay

The amount of compensation for each region equals to their willingness to pay. Therefore, the total pollution reduction will be 5 to make the marginal saving for reduction cost equal the marginal compensation.

The compensation received by Region O is $S_{\Delta}ADE = 32$ while the compensation received by Region R is $S_{\Delta}ABC = 1$.

If the payments were placed in the general office of the UN, the outcome would not be any different from an efficiency point of view because the allocation does not affect the polluter's behavior.

(c)

In region O, let $13 - Q = 10$, we have $Q_O = 3$. In region R, let $12 - 2Q = 10$, we have $Q_R = 1$. Therefore the total reduction is 4. Under proposal B, the polluters negotiate their pollution reduction in local region independently. Therefore, they will not consider their damage to the other region, which causes a lower total reduction.

8. (6.4)

(a)

$$NPV = -100 + \frac{1,000,000}{(1+10\%)^{100}} = -27.43 < 0$$

Therefore, this investment is a bad idea if $r = 10\%$.

(b)

$$NPV = -100 + \frac{1,000,000}{(1+1\%)^{100}} = 369,611.2 > 0$$

Therefore, this investment is a good idea if $r = 1\%$.

(c)

Suppose we will pay p dollars which should satisfies

$$-p + \frac{1,000,000}{(1+2\%)^{100}} \geq 0$$
$$\Rightarrow p \leq 138,033$$

9. (6.5)

(a)

Tourists and hotel owners will be negatively affected by the tax because it will increase the price for tourists and therefore decrease the demand for rooms.

(b)

The tourists, hotel owners, and the residents of Matildastan will be positively affected by the cleanup. Because of the cleanup, the tourists will get a better view; the residents of Matildastan will have a better living environment; the hotel owner in Matildastan will receive more guests.

(c)

Assume the market is competitive and the demand function is linear. Therefore, the room price in both hotel should be the same. Since the variable cost of a room is $\$30$, there will be no supply below this price.

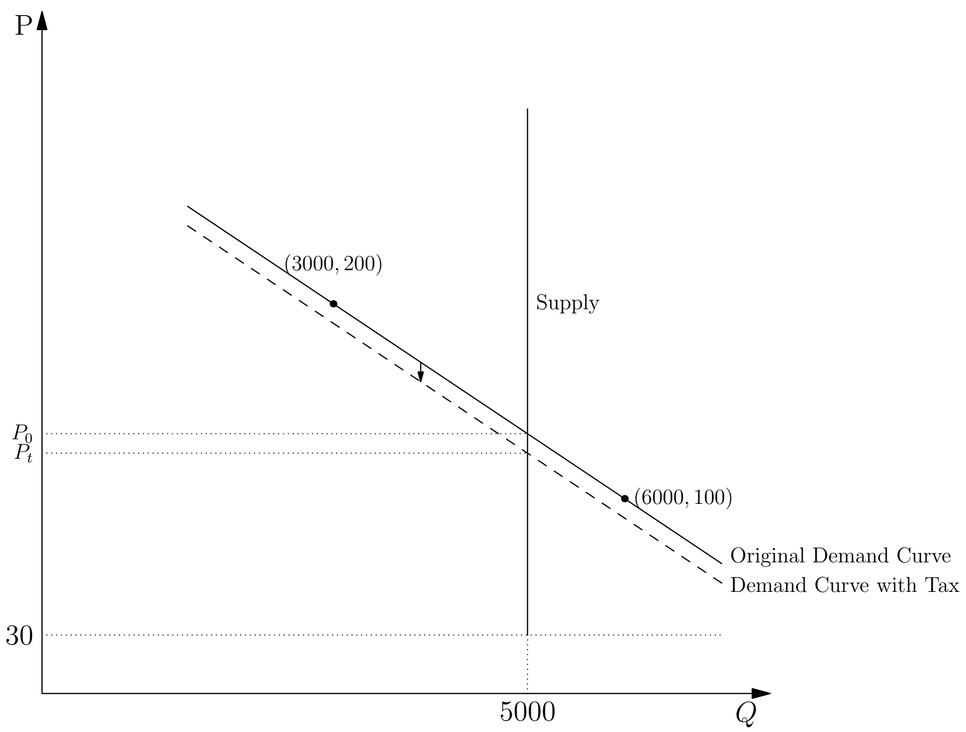

Figure 9: Total Demand and Supply under Room Tax

According to Figure 9, under the tax, the room price will decrease \$10 from P_0 to P_t

(d)

See Figure 10.

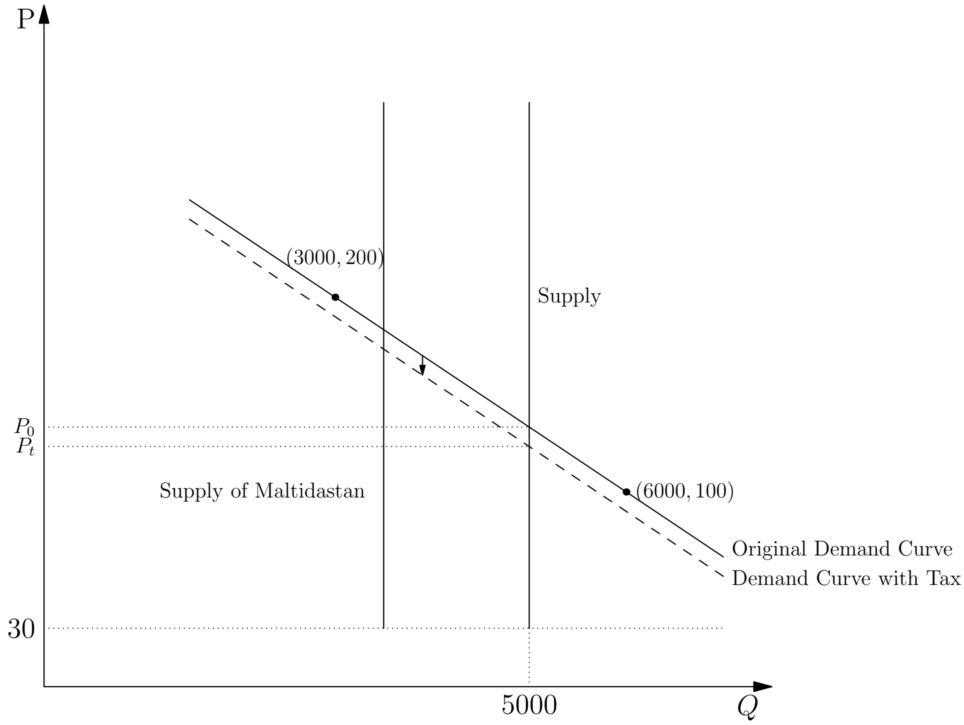

Figure 10: Total Demand and Supply under Room Tax only in Maltidastan

(e)

Since the supply is inelastic and labor prices are set in a larger regional market, hotel owners cannot transfer the tax on them. Therefore, they ultimately pay for the tax.