# Solution to Problem Set 1

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1.

| Price | A | B | Aggregate Demand |
|---|---|---|---|
| 0 | 9 | 6 | 15 |
| 1 | 6 | 5 | 11 |
| 2 | 4 | 3 | 7 |
| 4 | 2 | 1 | 3 |
| 5 | 0 | 0 | 0 |

2.

| $P$ | $Q_A$ | $Q_B$ | $Q$ | $\overline{P}$ | $\overline{Q_A}$ | $\overline{Q_B}$ | $\overline{Q}$ | $\Delta P$ | $\Delta Q_A$ | $\Delta Q_B$ | $\Delta Q$ | $\epsilon_{d,p}^A$ | $\epsilon_{d,p}^B$ | $\epsilon_{d,p}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 9 | 6 | 15 | | | | | | | | | | | |
| 1 | 6 | 5 | 11 | 0.5 | 7.5 | 5.5 | 13 | 1 | -3 | -1 | -4 | -0.20 | -0.09 | -0.15 |
| 2 | 4 | 3 | 7 | 1.5 | 5 | 4 | 9 | 1 | -2 | -2 | -4 | -0.60 | -0.75 | -0.67 |
| 4 | 2 | 1 | 3 | 3 | 3 | 2 | 5 | 2 | -2 | -2 | -4 | -1.00 | -1.50 | -1.20 |
| 5 | 0 | 0 | 0 | 4.5 | 1 | 0.5 | 1.5 | 1 | -2 | -1 | -3 | -9.00 | -9.00 | -9.00 |

3.

Since $\epsilon_{d,p} = \frac{\Delta Q/Q}{\Delta P/P}$, we have $\frac{\Delta Q}{Q} = \epsilon_{d,p} \times \frac{\Delta P}{P} = -0.25 \times 10\% = -2.5\%$. That is to say, the quantity demanded would fall 2.5% if the price were to rise 10%.

4.

(a) When $P = 5$, $Y = 100$ and $P_a = 4$, we have

$$Q = 4 - 2P + 2Y + 3P_a = 4 - 10 + 200 + 12 = 206$$

(b)

$$\begin{aligned}
\epsilon_{d,p} &= \frac{P}{Q^d}\frac{\partial Q^d}{\partial P} = \frac{5}{206} \times (-2) = -0.0485 \\
\epsilon_{d,Y} &= \frac{Y}{Q^d}\frac{\partial Q^d}{\partial Y} = \frac{100}{206} \times 2 = 0.9709 \\
\epsilon_{d,p_a} &= \frac{P_a}{Q^d}\frac{\partial Q^d}{\partial P_a} = \frac{4}{206} \times 3 = 0.0583
\end{aligned}$$

(c) Similar to (b), when $P = 4$, $Y = 200$ and $P_a = 5$, we have $Q = 411$.

$$\begin{aligned}
\epsilon_{d,p} &= -0.0195 \\
\epsilon_{d,Y} &= 0.9732 \\
\epsilon_{d,p_a} &= 0.0365
\end{aligned}$$

5.

Here, we should calculate the economic profit for Peter to start his own businues. If Peter continues working for the company, the total annual salary is $\$70,000$. His funds gives him additional $100,000 \times 2\% = \$2,000$ annual interest income. He can also obtain $\$20,000$ annual rent income from his second house. In summary, his opportunity cost for opening a restaurant is $\$92,000$.

If Peter starts the company, his accounting profit is $\$200,000 - \$90,000 = \$110,000$. (The start fund is sunk cost. You should not take it into consideration.)

The economic profit for Peter is $\$110,000 - \$92,000 = \$18,000 > 0$. Therefore, Peter should start his own business.

6.

$$\begin{aligned}
MC &= \frac{\partial TC}{\partial Q} = 5 + 4Q \\
ATC &= \frac{TC}{Q} = \frac{20}{Q} + 5 + 2Q \\
AFC &= \frac{FC}{Q} = \frac{20}{Q} \\
AVC &= \frac{VC}{Q} = 5 + 2Q
\end{aligned}$$

7.

(a)

$$P = 65 - Q$$

(b)

$$R = P \cdot Q = (65 - Q) \cdot Q = 65Q - Q^2$$
$$MR = \frac{\partial R}{\partial Q} = 65 - 2Q$$

(c)

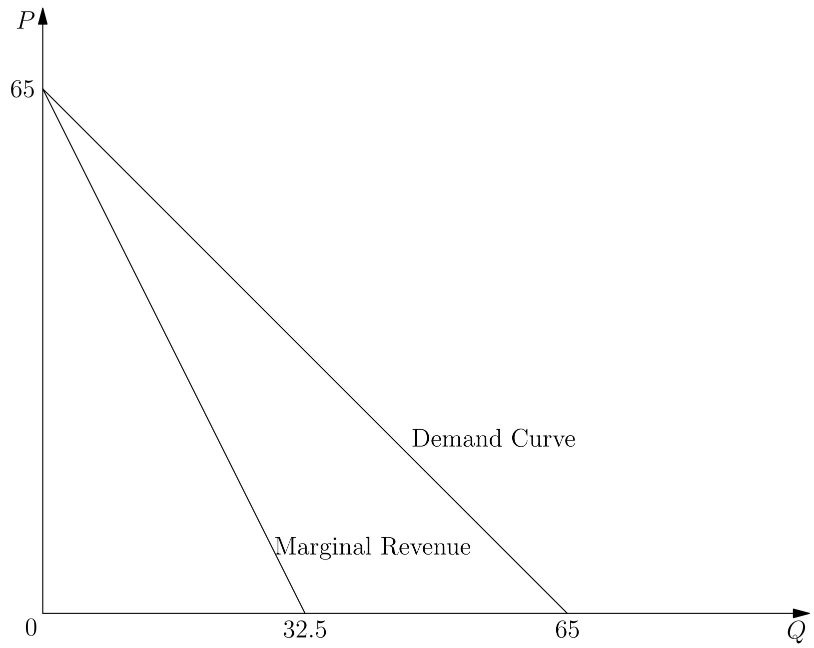


Figure 1: Demand Curve and MR Curve

(d) Firm's optimal output $Q^*$ satisfies $MR(Q^*) = MC(Q^*)$. Since $TC = 20 + 5Q + 2Q^2$, we have $MC = 5 + 4Q$. Solve the equation $MR = MC$, we have $Q^* = 10$.

(e) Firm's profit is $\Pi(Q) = R - TC = -3Q^2 + 60Q - 20$. When $Q^* = 10$, $\Pi^* = 280$.

8.

(a) At the competitive equilibrium, $Q^d = Q^s$. Solving this equation, we have $P^* = 1$, $Q^* = 2$ (See Figure 2).

(b) According to Figure 2, the yellow part is consumer surplus. Therefore, $CS = S_{\Delta PEP^*} = 2$.

(c) According to Figure 2, the red part is producer surplus. Therefore, $PS = S_{\Delta OEP^*} = 1$.

(d) With a tax $t = \$0.5$, producer receives $P^s$ while consumer pays $P^d = P^s + t$. Therefore, the new equilibrium quantity $Q^t$ satisfies $3 - Q = \frac{Q}{2} + 0.5$. Solving the equation, we have $Q^t = \frac{5}{3}$ and $P^d = \frac{4}{3}$.

(e) According to Figure 3, the yellow part is consumer surplus while the red part is producer surplus. Therefore, $CS = S_{\Delta PBP^d} = \frac{25}{18}$ and $PS = S_{\Delta OAP^s} = \frac{25}{36}$.

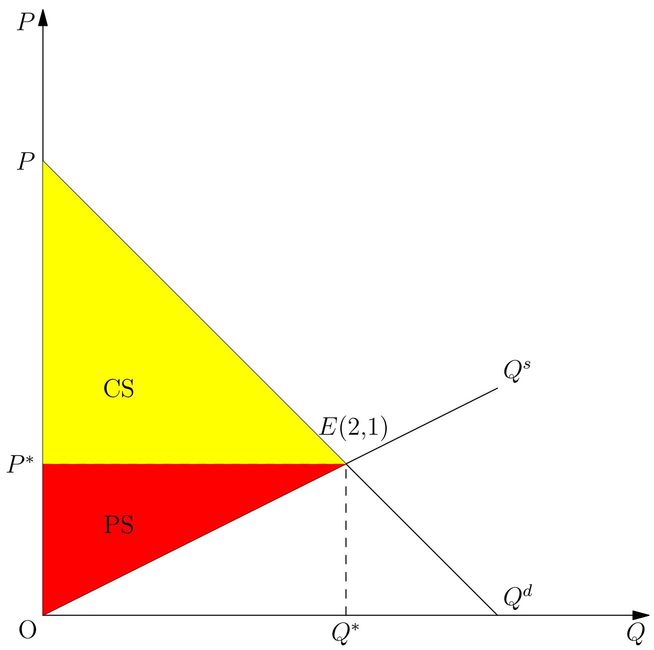


Figure 2: Competitive Equilibrium

(f) According to Figure 3, the blue part is the deadweight loss. Therefore, $DWL = S_{\Delta ABE^*} = \frac{1}{12}$.

9.

(a) Using the profit-maximizing condition, $P = MC$, the firm's supply function is

$$Q = 0.5P + 2$$

(b) When 40 such firm serve the market, the market supply function is

$$Q^{\text{market}} = 20P + 80$$

10. (a)

$$MC = 2Q$$
$$R = P \cdot Q = Q - \frac{1}{2}Q^2$$
$$MR = 1 - Q$$

(b) Let $MR = MC$, we have $Q^* = \frac{1}{3}$ and $P^* = \frac{5}{6}$.

(c) When $Q^* = \frac{1}{3}$, $\Pi^* = R^* - C^* = \frac{1}{6}$.

(d) According to Figure 4, the yellow part is consumer surplus while the red part is producer surplus. Therefore, $CS = \frac{1}{36}$ and $PS = \frac{1}{6}$.

(e) According to Figure 4, the blue part is the deadweight loss. Therefore, $DWL = \frac{1}{180}$.

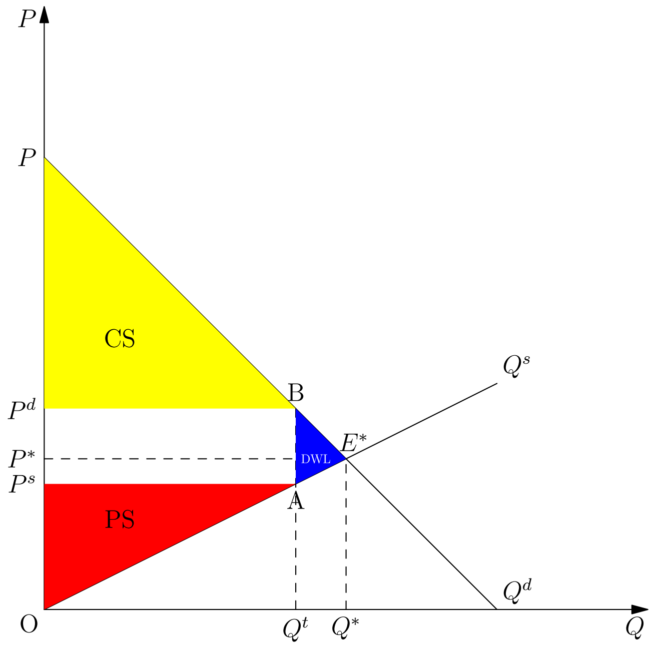


Figure 3: Equilibrium with Tax

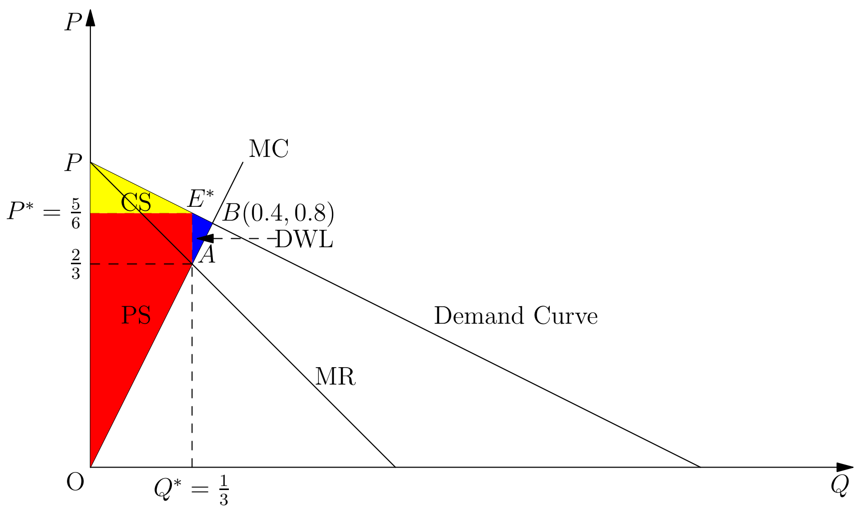


Figure 4: Monopoly