

# ENVIRON 805K Problem Set 1

Each sub-question is worth of 10 points.

1. Below is a demand schedule for goods A and B. Please derive the aggregate demand.

Price	A	B
0	9	6
1	6	5
2	4	3
4	2	1
5	0	0

2. Arc Elasticities: using the demand schedule in Q1, derive the price elasticity of demand for A, B and aggregate demand. Please derive the elasticities step by step not just the one between the first and last point.
3. If Winnie-the-Pooh's price elasticity of demand for honey is  $-0.25$ , how much would the quantity demanded fall if the price were to rise 10%?
4. Point Elasticities: suppose that the demand function for iPhone 6 is described by

$$Q = 4 - 2P + 2Y + 3P_a$$

In this form  $Q$  is quantity,  $P$  is iPhone price,  $Y$  is income and  $P_a$  is Android phone price. Assume that  $P = 5$ ,  $Y = 100$ ,  $P_a = 4$ . Please answer the following questions.

- (a) Trivial: What is the quantity demanded for iPhone 6?
  - (b) Please derive the price, income and cross-price elasticities.
  - (c) Please derive three elasticities under new parameters:  $P = 4$ ,  $Y = 200$ ,  $P_a = 5$
5. Peter is working for a company with an annual salary at \$70,000. Since Peter loves pizza and beer so much that he decides to start his own business that produces both goods. The estimated annual revenue is \$200,000 and annual explicit expense is \$90,000. He does not pay his own salary. He has \$100,000 personal funds to start the business. The market annual interest rate is 2%. He uses his own second house for production. The annual rent is \$20,000 on the market. Shall Peter start his own business? Explain.
  6. A firm's short-run cost function is given by  $TC = 20 + 5Q + 2Q^2$ , please derive the firm's short-run marginal cost, average total cost, average fixed cost, average variable cost.
  7. A firm's short-run cost function is given by  $TC = 20 + 5Q + 2Q^2$ . The firm faces a demand function given by  $Q = 65 - P$ . Please answer the following questions:
    - (a) Derive the firm's inverse demand function.
    - (b) Derive the firm's marginal revenue function.
    - (c) Show demand curve and marginal revenue curve graphically.
    - (d) What is the firm's optimal output?
    - (e) Calculate the maximum profit that the firm can achieve.

8. In a perfectly competitive market, the demand function is  $Q^d = 3 - P$  and the supply function is  $Q^s = 2P$ . Answer the following questions. Show your results both numerically and graphically.
- What is the competitive equilibrium?
  - What is the consumer surplus?
  - What is the producer surplus?
  - If the government imposes a tax of \$0.5 per unit, what are the new equilibrium quantity and price?
  - What are the consumer surplus and producer surplus with the above tax?
  - What is the deadweight loss caused by the above tax?
9. Suppose the cost function for the typical firm in a perfectly competitive market is  $C(Q) = 25 - 4Q + Q^2$ . Forty such firms in the market. Please answer the following question:
- What is the firm's supply function?
  - What is the market supply function?
10. In a pure monopoly market, the demand function is  $Q = 2 - 2P$  and the monopolist's cost function is  $C = Q^2$ . Answer the following questions both numerically and graphically.
- What is the marginal cost and marginal revenue?
  - What is the equilibrium quantity and price under monopoly?
  - What is the maximum profit for the monopolist?
  - What are the consumer surplus and producer surplus?
  - What is the deadweight loss?